

Supersymmetry and stability of flux vacua

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ABSTRACT: We describe a modified KKLT mechanism of moduli stabilization in a supersymmetric Minkowski vacuum state. In this mechanism, supersymmetry ensures vacuum stability and positivity of the mass matrix for the dilaton, complex structure, and volume modulus.

KEYWORDS: Flux compactifications, Superstring Vacua, dS vacua in string theory.

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1. Introduction

In the last few years, there has been a significant progress in solving the problem of moduli stabilization in string theory. We should mention, in particular, a mechanism for stabilizing the dilaton and the complex structure moduli by fluxes [1] and the KKLT construction, which allows, in principle, to fix all remaining moduli, including the volume modulus describing the size of the compactified space [2].

The standard way towards moduli stabilization in the KKLT construction consisted of two parts. One should first fix the axion-dilaton and the complex structure moduli by turning on fluxes following [1]. In order to do this, one should find a minimum of the potential in the effective N=1 supergravity theory with a superpotential given by

$$W_{\text{flux}} = \int G_3 \wedge \Omega = f \cdot \Pi(x) - \tau h \cdot \Pi(x). \tag{1.1}$$

Here f is a flux vector from the RR 3-form, τ is the axion-dilaton, h is a flux vector from the NS form and Π is a period vector depending on complex structure moduli x .

Once the moduli τ and x are fixed at some values τ_0 and x_0 , one should fix the volume modulus by adding to the superpotential a term $W_{\text{np}}(\rho)$ describing nonperturbative effects (e.g. gluino condensation). The simplest nonperturbative term of this type can be written as

$$W_{\text{np}} = Ae^{-a\rho}. \tag{1.2}$$

In practice, the procedure of finding a minimum of the scalar potential $V(\tau, x, \rho)$ was replaced by a simpler procedure of finding its supersymmetric extremum and making sure that this extremum is a minimum with respect to the volume modulus.

However, this is not the end of the story: The supersymmetric extremum of the potential in N=1 supergravity usually occurs at negative values of the potential (AdS extremum). One must then uplift this minimum to make the value of the potential slightly positive at

that point, so that $V_0 \sim 10^{-120}$ in Planck units. The uplifting is achieved by adding to the potential a term $\frac{C}{\rho^2}$ corresponding to the energy of the $\overline{D3}$ brane [2, 4] or $D7$ brane [3]. Some models of flux stabilization lead directly to dS vacua [5].

Properties of the potential after stabilization of the axion-dilaton and complex structure moduli were studied in [6]. The authors found that a certain fraction of all AdS extrema corresponds to saddle points. This is not a problem in AdS because a supersymmetric extremum in AdS can be stable even if it is a maximum or a saddle point [7]. However, after the uplifting, AdS space becomes dS space, and the fields near a maximum or a saddle point of the scalar potential become unstable.

This is not a real problem since a considerable fraction of all AdS vacua studied in [6] are in fact minima. However, the problem may reappear again when one tries to stabilize all fields including the volume modulus: Even if the dilaton and volume modulus may seem stabilized, one may encounter instabilities with respect to their simultaneous evolution [8, 9]. The root of the problem is the same: One should carefully check whether the AdS extremum prior to the uplifting is a minimum but not a saddle point with respect to all moduli.

Another set of problems appears when one tries to analyze a possible impact of the uplifting term $\frac{C}{\rho^2}$ on the properties of the extremum. A nice property of the original KKLT model was that the nonperturbative potential induced by the superpotential (1.2) was very sharp, so an addition of the slowly decreasing function $\frac{C}{\rho^2}$ did not destabilize the minimum. However, it changed the position of the field ρ . For some other nonperturbative potentials, uplifting may lead to destabilization, either with respect to the volume modulus or with respect to some other moduli. So in general one should make an additional investigation to check that the extremum of the effective potential remains a minimum after the uplifting.

Over the last few years many examples of all moduli stabilization in type IIB theory have been constructed [10]. Many of them show that in fairly generic examples of the KKLT type, one can find explicit class of models with de Sitter vacua which are tachyon free and the potential is a local minimum in all directions. These examples support the general statistical analysis in [6] which argues that there are many examples without tachyons. However, one does have to check each new model and verify that it is tachyon-free with respect to all moduli.

The main idea of our paper is to stabilize all moduli in a supersymmetric Minkowski vacuum, extending the approach of ref. [11]. As we will show, the absence of tachyons in such models is guaranteed by construction.

This issue is closely related to the problem of supersymmetry breaking in the KKLT scenario. Different aspects of this problem were studied by many authors; see e.g. [12] and references therein. One feature of supersymmetry breaking is particularly important in the cosmological context. In the original version of the KKLT scenario, the height of the barrier separating the minimum of the potential from the 10D Minkowski vacuum is $O(m_{3/2}^2)$. If one tries to achieve inflation by adding to the KKLT potential a contribution of the inflaton field, it always comes divided by a certain degree of ρ . Therefore adding a large inflaton contribution may destabilize the volume modulus. One can show that the barrier disappears and the universe decompactifies if the energy density of the inflaton field

(or of any other fields living in 4D) becomes much greater than the height of the barrier. This implies that the KKLT minimum appears only very late in the cosmological evolution, when the Hubble constant becomes smaller than the gravitino mass [11].

There are some ways to slow down the rolling of the volume modulus at the first stages of the evolution of the universe and avoid decompactification, see e.g. [13, 14]. However, for many years it was believed that the gravitino mass is extremely small, $m_{3/2} = O(1)\text{TeV} \sim 10^{-15}M_p$. In this case one would come to an unexpected conclusion that the vacuum stabilization in string theory is a very low energy scale phenomenon. This would also mean that the Hubble constant during inflation cannot be greater than $m_{3/2} \sim 10^{-15}M_p$. Whereas such low-scale inflationary models are possible, they are highly unconventional and require introduction of several extremely small parameters.

There are two ways to alleviate this problem. First of all, one may try to construct the models with $m_{3/2} \gg 10^{-15}M_p$, see e.g. [15–17]. Another approach is to modify the original KKLT mechanism [5, 11, 17]. In this paper we will study this possibility following ref. [11]. It was shown there that if one considers racetrack superpotentials with two exponents, $W_{\text{np}} = Ae^{-a\rho} + Be^{-b\rho}$, one can find such parameters that the scalar potential has a supersymmetric minimum at $V_0 = 0$, i.e. in Minkowski space, even before an uplifting. In this approximation, the gravitino mass vanishes. By allowing a small supersymmetry breaking and using a very small uplifting, one can make the height of the barrier stabilizing the KKLT minimum, as well as the mass squared of the volume modulus, many orders of magnitude greater than $m_{3/2}^2$. This allows inflation at $H \gg m_{3/2}$.

In the present paper we will show that the same mechanism allows to alleviate the vacuum instability problems discussed above. The reason is very simple: if, following [11], we find a local supersymmetric extremum of the scalar potential *in Minkowski space*, then the corresponding extremum will be stable in Minkowski space due to the positive energy theorem in supergravity [18]. We will demonstrate this explicitly by the investigation of the properties of the effective potential near the supersymmetric extremum; for a similar approach see also [19].

Of course, we still need to break supersymmetry and slightly uplift the potential to $V_0 \sim 10^{-120}$. However, this should not destabilize the potential if the gravitino mass is much smaller than the masses of the stabilized moduli.

2. Convexity of the F-term potential in supersymmetric Minkowski vacua

It will be useful for our purposes to present here the general compact expression for the second derivative of the F-term potential in N=1 supergravity in terms of the fermion masses in the supersymmetric vacuum. We will consider the most general case of an arbitrary number of fields z_a , arbitrary Kähler potential $\mathcal{K}(z_a, \bar{z}_a)$ and arbitrary holomorphic superpotential $W(z_a)$.

We use the notation close to [20] where the N=1 supergravity action with fermionic terms is presented in a form useful for the study of the vacua of the effective theory. We use units in which $M_P = 1$. In addition to the holomorphic superpotential $W(z_a)$, we define a

covariantly holomorphic complex gravitino mass as,

$$e^{K/2}W \equiv m(z_a, \bar{z}_{\bar{a}}), \quad (2.1)$$

which is related to the (real) gravitino mass,

$$M_{3/2}^2 = |m \bar{m}|. \quad (2.2)$$

The complex gravitino mass is covariantly holomorphic, which means that

$$\bar{D}_{\bar{a}}m \equiv \left(\partial_{\bar{a}} - \frac{1}{2}K_{\bar{a}} \right) m = 0, \quad D_a \bar{m} \equiv \left(\partial_a - \frac{1}{2}K_a \right) \bar{m} = 0. \quad (2.3)$$

We also define the Kähler covariant derivatives as,

$$D_a m = \partial_a m + \frac{1}{2}(\partial_a K)m \equiv m_a, \quad \bar{D}_{\bar{a}} \bar{m} = \partial_{\bar{a}} \bar{m} + \frac{1}{2}(\partial_{\bar{a}} K)\bar{m} \equiv \bar{m}_{\bar{a}}. \quad (2.4)$$

The complex masses of the chiral fermions in N=1 supergravity are equal to

$$D_a D_b m \equiv m_{ab}, \quad \bar{D}_{\bar{a}} \bar{D}_{\bar{b}} \bar{m} \equiv \bar{m}_{\bar{a}\bar{b}}. \quad (2.5)$$

The Kähler metric is $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$. Using this notation we can rewrite the standard F-term potential $V = e^{\mathcal{K}}(|DW|^2 - 3|W|^2)$ as follows

$$V = m_a g^{a\bar{b}} \bar{m}_{\bar{b}} - 3|m|^2 \equiv |m_a|^2 - 3|m|^2. \quad (2.6)$$

The first derivative of the potential becomes,

$$\partial_a V = D_a V = -2m_a \bar{m} + m_{ab} g^{b\bar{c}} \bar{m}_{\bar{c}}. \quad (2.7)$$

where we have taken into account that $D_a \bar{m}_{\bar{b}} = g_{a\bar{b}} \bar{m}$ and that the potential is Kähler invariant.

From now on we focus only on supersymmetric vacua, i.e.,

$$D_a m \equiv m_a = 0, \quad \bar{D}_{\bar{a}} \bar{m} \equiv \bar{m}_{\bar{a}} = 0, \quad \Rightarrow \quad \partial_a V = 0. \quad (2.8)$$

In this case, the second derivatives of the potential at $\partial_a V = 0$ are given by

$$\partial_{\bar{b}} \partial_a V = -m_{ba} \bar{m}, \quad \partial_{\bar{b}} \partial_{\bar{a}} V = -\bar{m}_{\bar{b}\bar{a}} m, \quad (2.9)$$

$$\partial_{\bar{b}} \partial_a V = -2g_{ba} m \bar{m} + m_{ac} g^{c\bar{c}} \bar{m}_{\bar{c}\bar{b}}. \quad (2.10)$$

In general we see that the AdS critical point is a saddle point since there are positive and negative directions at $m \neq 0$. That is why uplifting of the supersymmetric AdS extremum may lead to an instability. This instability is not unavoidable though. In particular, it disappears if the complex gravitino mass \bar{m} is much smaller than all moduli masses.

This fact becomes manifest when the minimum of the potential occurs at vanishing value of the cosmological constant. In this case, one has $m = 0$ and $m_a = 0$ at the supersymmetric critical point, so we can find the following matrix \mathcal{M} ,

$$\mathcal{M}_{ba} = \partial_b \partial_a V = 0, \quad \mathcal{M}_{\bar{b}\bar{a}} = \partial_{\bar{b}} \partial_{\bar{a}} V = 0, \quad (2.11)$$

$$\mathcal{M}_{a\bar{b}} = \partial_a \partial_{\bar{b}} V = m_{ac} g^{c\bar{c}} \bar{m}_{\bar{c}\bar{b}}. \quad (2.12)$$

This is a positive definite matrix, and consequently all its eigenvalues are positive. Indeed, for an arbitrary vector ϕ ,

$$\phi_a \mathcal{M}_{a\bar{b}} \bar{\phi}_{\bar{b}} = \phi_a m_{ac} g^{c\bar{c}} \bar{m}_{\bar{c}\bar{b}} \bar{\phi}_{\bar{b}} = \Phi_c g^{c\bar{c}} \bar{\Phi}_{\bar{c}}, \quad (2.13)$$

where $\Phi_c = \phi_a m_{ac}$. We now recall that the matrix $g^{c\bar{c}}$ is the inverse matrix of the Kähler metric and therefore is required to be positive definite, so we have $\Phi_c g^{c\bar{c}} \bar{\Phi}_{\bar{c}} \geq 0$ for any vector Φ . Thus \mathcal{M} is a positive definite matrix, namely,

$$\phi_a \mathcal{M}_{a\bar{b}} \bar{\phi}_{\bar{b}} \geq 0. \quad (2.14)$$

This is true for an arbitrary vector ϕ_a , which means that the second derivative of the potential is non-negative in all directions in the moduli space.

One can easily understand the meaning of this result in terms of equation (2.6): $V = |m_a|^2 - 3|m|^2$. In a supersymmetric extremum with $V = 0$, the function m is of the second order in the deviation from this extremum δz , because both m and its first derivative m_a vanish at the extremum. Therefore the negative term $-3|m|^2$ is quartic with respect to δz ; the only term quadratic in δz is the non-negative term $|m_a|^2$.

One potential caveat in this discussion is the possibility that the mass matrix may vanish in some directions. This may occur, e.g., if the potential is quartic in δz . However, one can generalize the argument given above, expanding m in all higher powers of δz , and show that m_a always contains terms lower order in δz as compared with m . Therefore at small δz the positive term $|m_a|^2$ dominates and the potential is convex. A similar argument can be found in [21].

The only exception from this rule may occur if all higher order terms in expansion of m in terms of δz vanish in some direction, which corresponds to a flat direction of the potential. However, one often finds (as we will find in an example to be considered in Section 4), that the solutions of equations $m = 0$ and $m_a = 0$ represent a discrete set of points instead of a line of supersymmetric points. In this case one can show that the potential cannot have flat directions, so all moduli are indeed stabilized. To prove it, one may assume that the potential has a flat direction along which $V = |m_a|^2 - 3|m|^2 = 0$. This condition can be considered as a differential equation on m with the initial condition $m = 0$, $m_a = 0$, for all a . One can easily check that the only solution of this equation along the flat direction is $m = 0$, $m_a = 0$, which means that supersymmetry should take place along the valley. In other words, the potential has flat directions only if the supersymmetric extrema form a valley. If the supersymmetric solution represents a discrete set of points, each of them represents an isolated *minimum* of the potential.

Of course, we do not live exactly in Minkowski space, and supersymmetry in our world is broken. In the simplest KKLT models the gravitino mass \bar{m} was of the same order of magnitude as the mass of the volume modulus [11]. The main advantage of the modified version of this mechanism, which was proposed in [11] and which we are going to generalize below, is that the gravitino mass \bar{m} in this scenario can be many orders of magnitude smaller than the masses of all other fields in eqs. (2.9), (2.10). For example, all parameters of the modified KKLT construction may have GUT scale or stringy scale, whereas the gravitino

mass may be 15 orders of magnitude smaller. In this case our eqs. (2.9), (2.10) imply that a small supersymmetry breaking should not affect the positive definiteness of the moduli mass matrix. This is the main idea of our work.

3. Supersymmetric 4d Minkowski vacua of IIB string theory

Now let us discuss how one may find the supersymmetric extrema of the potential with respect to all moduli.

Consider a toy model superpotential for moduli stabilization where in addition to the axion-dilaton field τ there are some complex structure fields x_i and a total volume ρ ,

$$W(\tau, x, \rho) = W_{\text{flux}}(\tau, x) + W_{\text{np}}(\rho), \tag{3.1}$$

where, as in eq. (1.1),

$$W_{\text{flux}} = \int G_3 \wedge \Omega = f \cdot \Pi(x) - \tau h \cdot \Pi(x). \tag{3.2}$$

Following [11], we add to it the racetrack superpotential $W_{\text{np}} = Ae^{-a\rho} + Be^{-b\rho}$ with $a = \frac{2\pi}{N}$, $b = \frac{2\pi}{M}$, where N and M are some integers [22]. The total superpotential is given by

$$W = f \cdot \Pi(x) - \tau h \cdot \Pi(x) + Ae^{-a\rho} + Be^{-b\rho}. \tag{3.3}$$

Assume that the potential has a supersymmetric Minkowski minimum at ρ_0 , τ_0 , and x_0 . We therefore have that at this minimum

$$W = 0, \quad \partial_\rho W = \partial_i W = \partial_\tau W = 0. \tag{3.4}$$

This set of equations resembles the ones obtained in [23] for flux vacua of type IIB string theory, but there is an important difference. The models considered in [23] did not include the volume modulus, only τ and x , which can be stabilized by fluxes. Furthermore they considered two types of models, ones with $W_{\text{flux}}(\tau, x)$ vanishing at the critical point, $W_0 \equiv W_{\text{flux}}(\tau_0, x_0) = 0$, and others where $W_0 \neq 0$. Correspondingly, there were two possible sets of equations:

$$W_0 = 0, \quad \partial_i W_{\text{flux}} = 0, \quad \partial_\tau W_{\text{flux}} = 0, \tag{3.5}$$

or

$$W_0 \neq 0, \quad D_i W_{\text{flux}} = 0, \quad D_\tau W_{\text{flux}} = 0. \tag{3.6}$$

Meanwhile, in our case the total superpotential $W(\tau, x, \rho) = W_{\text{flux}}(\tau, x) + W_{\text{np}}(\rho)$ vanishes at the supersymmetric critical point, and covariant derivatives coincide with simple derivatives even for $W_0 \equiv W_{\text{flux}}(\tau_0, x_0) \neq 0$:

$$W_0 \neq 0, \quad \partial_i W_{\text{flux}} = 0, \quad \partial_\tau W_{\text{flux}} = 0. \tag{3.7}$$

In the minimum with respect to x^i and τ , the superpotential $W_{\text{flux}}(\tau, x) + W_{\text{np}}(\rho)$ acquires the familiar racetrack form [22]

$$W = W_0 + Ae^{-a\rho} + Be^{-b\rho}. \tag{3.8}$$

Equation $\partial_\rho W = 0$ in this theory has a following solution [11]:

$$\rho_0 = \frac{1}{a-b} \ln \left| \frac{aA}{bB} \right|. \quad (3.9)$$

The remaining equations look as follows:

$$\partial_\tau W = 0 \quad \Rightarrow \quad h \cdot \Pi(x_0) = 0. \quad (3.10)$$

Therefore

$$W_0 \equiv W_{\text{flux}}(\tau_0, x_0) = f \cdot \Pi(x_0). \quad (3.11)$$

The next set of equations is

$$\partial_i W = 0 \quad \Rightarrow \quad f \cdot \Pi_{,i}(x_0) - \tau_0 h \cdot \Pi_{,i}(x_0) = 0 \quad (3.12)$$

which defines the axion-dilaton values at the minimum,

$$\tau_0 = \frac{f \cdot \Pi_{,i}(x_0)}{h \cdot \Pi_{,i}(x_0)}. \quad (3.13)$$

Finally, the requirement $W(\tau_0, x_0, \rho_0) = 0$ leads to a condition

$$W_0 = -Ae^{-a\rho_0} - Be^{-b\rho_0} = -A \left| \frac{aA}{bB} \right|^{\frac{a}{b-a}} - B \left| \frac{aA}{bB} \right|^{\frac{b}{b-a}}. \quad (3.14)$$

One way to solve this set of equations is to find the solutions for x_0 and τ_0 using eqs. (3.10), (3.13), then calculate W_0 using eq. (3.11), and finally find a set of parameters A , B , a and b such that $W_0 = -A \left| \frac{aA}{bB} \right|^{\frac{a}{b-a}} - B \left| \frac{aA}{bB} \right|^{\frac{b}{b-a}}$.

4. An example of the solution

To be sure that the mechanism proposed above can actually work, we will construct a particular example using the results of refs. [11, 23]. We will consider the model A of ref. [23] with one complex structure modulus x_1 , which, following notation in [23] we will denote by ψ . We will solve our equations expanding them in series in ψ^2 , which is supposed to be small near the extremum.

According to [23], at small ψ , the period vector in their model can be expanded as

$$\Pi_i = c_0 p_0 + c_2 p_2 \psi^2 + c_4 p_4 \psi^4 + \dots \quad (4.1)$$

Here the vectors p_k are given by

$$p_k = m_i \cdot \tilde{p}_k, \quad m_i = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \quad (4.2)$$

with

$$\tilde{p}_k^T = \left(\alpha^{2(k+1)}, \alpha^{(k+1)}, 1, \alpha^{7(k+1)} \right), \quad \alpha = e^{i\frac{\pi}{4}} \quad (4.3)$$

and

$$\begin{aligned}
 c_0 &= (2\pi i)^3 \frac{\sqrt{\pi}}{2\Gamma^4(7/8)} \frac{\exp(\frac{7\pi i}{8})}{\sin(\frac{\pi}{8})}, \\
 c_2 &= -(2\pi i)^3 \frac{2\sqrt{\pi}}{\Gamma^4(5/8)} \frac{\exp(\frac{5\pi i}{8})}{\sin(\frac{3\pi}{8})}, \\
 c_4 &= (2\pi i)^3 \frac{4\sqrt{\pi}}{\Gamma^4(3/8)} \frac{\exp(\frac{3\pi i}{8})}{\sin(\frac{5\pi}{8})}.
 \end{aligned}
 \tag{4.4}$$

The solutions we will present here will be valid for small $|\psi|$, where the ψ modulus will be stabilized in a self-consistent approximation.

We first find τ_0 , ψ_0 and W_0 using eqs. (3.13), (3.10) and (3.11) for a particular set of parameters used in ref. [23] namely: $f = \{-28, 24, 7, -24\}$, $h = \{-34, 41, 12, -17\}$. Note that it was shown in [23] that, at the lowest order in ψ^2 , the moduli space point where $\partial_i W_{\text{flux}} = 0$ and $\partial_\tau W_{\text{flux}} = 0$ one finds $W_0 = 0$. If, however, one solves these equations more precisely, at second order in ψ^2 , one finds a small but nonzero value of W_0 .

In [23] it was necessary to solve a system of equations $D_\psi W_{\text{flux}} = 0$, $D_\tau W_{\text{flux}} = 0$, each of which depended on W_0 . However, as we already discussed in the previous section, in our model one should solve a simpler set of equations $\partial_\psi W_{\text{flux}} = 0$, $\partial_\tau W_{\text{flux}} = 0$. After finding τ_0 and ψ_0 , one can calculate W_0 using eq. (3.11).

Our investigation gives the following results for τ_0 , ψ_0 and W_0 :

$$\psi_0 \approx 0.088 - 0.036i, \quad \tau_0 \approx 1.4i, \quad W_0 \approx 0.0766 - 0.1849i, \quad |W_0| \approx 0.2.
 \tag{4.5}$$

The only remaining problem is to find parameters A , B , a and b such that $W_0 = -A \left| \frac{aA}{bB} \right|^{\frac{a}{b-a}} - B \left| \frac{aA}{bB} \right|^{\frac{b}{b-a}}$. Obviously, there are many sets of parameters satisfying this condition. One particular set is

$$a = \frac{2\pi}{200}, \quad b = \frac{2\pi}{100}, \quad A = -\beta, \quad B = 10\beta,
 \tag{4.6}$$

where $\beta = 3.06 - 7.39i$. Fig. 1 shows the shape of the potential for the volume modulus for this set of parameters.

We plot the slice of the total potential

$$V(\tau, \bar{\tau}; \psi, \bar{\psi}; \rho, \bar{\rho}) = e^{K(\tau, \bar{\tau}; \psi, \bar{\psi})} e^{K(\rho, \bar{\rho})} (|D_\tau W|^2 + |D_\psi W|^2 + |D_\rho W|^2 - 3|W|^2)
 \tag{4.7}$$

at the critical values of τ_0 and ψ_0 given in eq. (4.5)

The scale of the potential is defined by the value of $e^{K(\tau, \bar{\tau}; \psi, \bar{\psi})} e^{K(\rho, \bar{\rho})} W_0$. It is interesting that in our example $e^{K(\tau, \bar{\tau}; \psi, \bar{\psi})}$ and $e^{K(\rho, \bar{\rho})}$ are of the same order. Therefore, when considering a truncated system where only ρ modulus is present, one has to use for effective W_0 the value of $e^K |W_{\text{flux}}|^2(f, h, \tau_0, \bar{\tau}_0; \psi_0, \bar{\psi}_0)$ as proposed in [23]. Indeed in our case $e^{K(\tau_0, \bar{\tau}_0; \psi_0, \bar{\psi}_0)} \sim 10^{-7}$ which is quite striking: it indicates that the Kähler potential due to complex structure moduli may become a dramatic source of a change of the scale of the potentials in various models.

The model presented above preserves all good features of the KKLT construction and has all moduli stabilized due to supersymmetry of the Minkowski vacuum state at $\rho_0 \approx 95$.

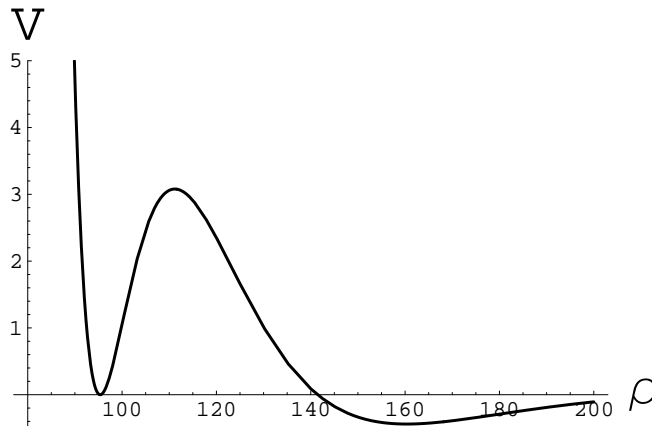


Figure 1: Potential of the volume modulus for $a = \frac{2\pi}{200}$, $b = \frac{2\pi}{100}$, $A = -\beta$, $B = 10\beta$, where $\beta = 3.06 - 7.39i$. The potential is shown in units of $10^{-15}M_p^4$.

5. Conclusions

The model described in this paper is somewhat more complicated than the original KKLT construction, but it has several advantages. First of all, the scale of supersymmetry breaking in this model can be hierarchically small as compared to stringy scale and to the scale of all moduli masses stabilized by the KKLT mechanism. This fact ensures vacuum stability, which otherwise would require a detailed study involving the investigation of convexity of the potential of all moduli, a rather daunting task with an uncertain outcome. Thus, at the very least, this model can be useful as a playground for investigation of the regions of the stringy landscape where the vacuum stability is protected by supersymmetry in Minkowski space. The smallness of supersymmetry breaking was previously postulated as a technical tool for solving the hierarchy problem and explaining smallness of the Higgs boson mass. It is quite intriguing that in string theory this condition can be linked to the requirement of vacuum stability.

It is equally interesting that the requirement of the hierarchically small supersymmetry breaking allows one to have inflation on a very high energy scale. This alleviates the problem of constructing unusual inflationary models with an extremely small scale of inflation. This also simplifies a solution of the problem of initial conditions for inflation, which is much easier to achieve for the high energy scale inflation.

In this paper we considered a model with the nonperturbative racetrack potential $Ae^{-a\rho} + Be^{-b\rho}$. However, the main idea of our work is more general: One should study all possible nonperturbative and perturbative corrections to the superpotential, and then try to find a supersymmetric state where the total superpotential (and, consequently, the total potential) vanishes. This problem is technically simpler than the more general problem of finding a supersymmetric AdS extremum because in Minkowski vacuum the covariant derivatives involving Kähler potential and superpotential reduce to the usual derivatives. Once the solution is found, it is automatically stable due to supersymmetry, the potential is convex, and this property should be preserved after the remaining small uplifting.

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